Calculus Warm Up #12-1

From Friday: $f(x) = \dfrac{3x}{\sqrt[3]{x^2+3}}$

List any extrema, points of inflection and describe the concavity of f.

$$f'(x) = \frac{x^2+9}{(x^2+3)^{4/3}}$$

$$f''(x) = \frac{-2x^3-54x}{3(x^2+3)^{7/3}}$$

Calculus Warm Up #12-1

From yesterday: $f(x) = \dfrac{3x}{\sqrt[3]{x^2+3}}$

List any extrema, points of inflection and describe the concavity of f.

$$f'(x) = \frac{x^2+9 \rightarrow \neq 0}{(x^2+3)^{4/3} \rightarrow \neq 0} \Big\}\ \text{So } \boxed{\text{no extrema}}$$

$$f''(x) = \frac{-2x^3-54x}{\underbrace{3(x^2+3)^{7/3}}_{\text{always } +}} \qquad \begin{aligned} &-2x^3-54x \longrightarrow 0 = -2x(x^2+27) \\ &3(x^2+3)^{7/3} \rightarrow \neq 0 \qquad x = 0 \text{ critical \#} \end{aligned}$$

f'': -1: $+$, 0, 1: $-$

$$\boxed{\begin{array}{l} \text{Point of inflection: } (0, 0) \\ \text{Concave up on } (-\infty, 0) \\ \text{Concave down on } (0, \infty) \end{array}}$$

HW Questions: p. 487

In Exercises 1–24, find dy/dx.

1. $y = \dfrac{\sin x}{x^2}$

3. $y = -x \tan x$

5. $y = \dfrac{1}{4}\sin 4x + x$

7. $y = \dfrac{1}{2}x - \dfrac{1}{4}\sin 2x$

$y' = \frac{1}{2} - \frac{1}{\cancel{4}\,2}(\cancel{2}\cos 2x)$

$\frac{1}{2}(1 - \cos 2x)$

9. $y = 2\csc^3(\sqrt{x})$ $\rightarrow$ $2(\csc\sqrt{x})^3$

11. $y = \tan\sqrt{1-x}$

13. $y = \tan(\underbrace{\arcsin x}_{\theta})$ $=$

15. $y = x \operatorname{arcsec} x$

$y = \dfrac{x}{\sqrt{1-x^2}}$

$y = x(1-x^2)^{-1/2}$

$y' = \dfrac{1}{(1-x^2)^{3/2}}$

17. $y = \underbrace{x(\arcsin x)^2}_{\text{prod. rule}} - 2x + \underbrace{2\sqrt{1-x^2}\arcsin x}_{\text{prod. rule}}$

$x \cdot 2(\arcsin x)^1\left(\frac{1}{\sqrt{1-x^2}}\right) + (1)(\arcsin x)^2 - 2 + 2\cancel{\sqrt{1-x^2}} \cdot \frac{1}{\cancel{\sqrt{1-x^2}}}$

$+ 2\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x)(\arcsin x)$

$\frac{2x \arcsin x}{\sqrt{1-x^2}} + (\arcsin x)^2 + 0 - \frac{2x \arcsin x}{\sqrt{1-x^2}}$

19. $y = \left(\frac{x^2+1}{2}\right)\arctan x$

21. $x = 2 + \sin y$

23. $\cos x^2 = xe^y$

19) $y' = \left(\frac{\cancel{x^2+1}}{2}\right)\frac{1}{\cancel{1+x^2}} + \frac{1}{\cancel{2}}(\cancel{2}x)\arctan x$

$\frac{1}{2} + x \arctan x$

19. $y = \left(\frac{x^2+1}{2}\right)\arctan x$

21. $x = 2 + \sin y$ →

23. $\cos x^2 = xe^y$

$$\frac{1}{\cos y} = \frac{\cancel{\cos y}\,\frac{dy}{dx}}{\cancel{\cos y}}$$

$$(-\sin x^2)(2x) = x\cdot e^y\frac{dy}{dx} + (1)e^y$$

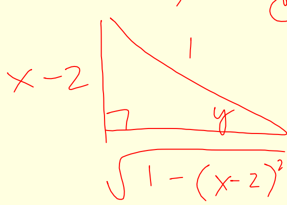
$$\frac{-2x\sin x^2 - e^y}{xe^y} = \frac{dy}{dx}$$

from original

$$\sin y = x-2$$

$$y = \arcsin(x-2)$$

$$\frac{dy}{dx} = \frac{1}{\cos(\underbrace{\arcsin(x-2)}_{y})}$$



In Exercises 25–30, find the second derivative of the function.

25. $f(x) = \cot x$

27. $h(x) = \frac{\cos x}{x}$ →

29. $f(x) = \arcsin 2x$

$$h(x) = x^{-1}(\cos x)$$

$$h'(x) = x^{-1}(-\sin x) + (-x^{-2})\cos x$$

$$= -x^{-2}(x\sin x + \cos x)$$

$$h''(x) = -x^{-2}\left[x\cos x + (1)\cancel{\sin x} - \cancel{\sin x}\right] + 2x^{-3}(x\sin x + \cos x)$$

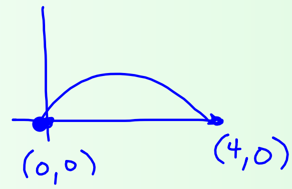
$$\frac{x}{x}\cdot\frac{-x\cos x}{x^2} + \frac{2x\sin x + 2\cos x}{x^3}$$

$$\frac{2x\sin x + 2\cos x - x^2\cos x}{x^3}$$

Parametric Equations:

Start with a relationship in 2 variables: $y = -x^2 + 4x$.
Let's say this equation models the path of an object.

$y = -x(x-4)$



We can talk about the position of the object along this path, but we need to add a third variable, say time, in order to talk about when the object might be at a certain place, (x, y), along the path.

In this case the additional parameter is time.

12.2 - Derivatives of parametric equations

Parametric Equations relate each variable to the parameter:

Example: Let t = the parameter,
both x and y are written as a function of t:

$x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4)$

We can find the rate of change between the 2 variables x and y, by differentiating with respect to t.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Is called Parametric Form of the Derivative

Note: $\frac{dx}{dt} \neq 0$

You can also take higher order derivatives!

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{\frac{d}{dt}\left[\frac{d^2y}{dx^2}\right]}{dx/dt}.$$

(from page 687-688)

Find slope and concavity at the point (2, 3), for the curve described by the parametric equations:

$x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4) \rightarrow y = \frac{t^2}{4} - 1$

$(t)^{1/2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{\frac{t}{2}}{\frac{1}{2}t^{-1/2}}$$

$$= \frac{\frac{t}{2}}{\frac{1}{2\sqrt{t}}}$$

$$= \frac{t}{2} \cdot \frac{2t^{1/2}}{1}$$

$$\frac{dy}{dx} = t^{3/2}$$

slope anywhere at time t.

Find slope and concavity at the point (2, 3), for the curve described by the parametric equations:

$x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

t? @ (2, 3)
x y

$2 = \sqrt{t} \rightarrow t = 4$

or

$3 = \frac{1}{4}(t^2 - 4)$

$12 = t^2 - 4$

$16 = t^2$

$4 = t$

$\frac{dy}{dx} = t^{3/2}$

so slope @ (2, 3)
when $t = 4$

$= (4)^{3/2}$

$= \boxed{8}$

Find slope and concavity at the point (2, 3), for the curve described by the parametric equations:

$x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$\frac{dy}{dx} = t^{3/2}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{\frac{d}{dt}\left[\frac{d^2y}{dx^2}\right]}{dx/dt}.$$

$$\frac{\frac{d}{dt}\left[t^{3/2}\right]}{\frac{1}{2\sqrt{t}}} = \frac{\frac{3}{2}t^{1/2}}{\frac{1}{2\sqrt{t}}}$$

$$= \frac{3\sqrt{t}}{\cancel{2}} \cdot \cancel{2}\sqrt{t}$$

$$\boxed{\frac{d^2y}{dx^2} = 3t}$$

@ (2, 3) where $t = 4$, $\frac{d^2y}{dx^2} = 12$

positive, so it is concave up there

Find the first and second derivatives and evaluate each at the given value of the parameter.

$x = \sqrt{t},\ y = 3t - 1 \qquad t = 1$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{\frac{d}{dt}(3t-1)}{\frac{d}{dt}(\sqrt{t})} = \frac{3}{\frac{1}{2\sqrt{t}}}$$

$$= 6\sqrt{t}$$

$$\boxed{@\ t=1 \quad \frac{dy}{dx} = 6}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[6\sqrt{t}\right]}{\frac{1}{2\sqrt{t}}}$$

$$= \frac{6 \cdot \frac{1}{2} t^{-1/2}}{\frac{1}{2\sqrt{t}}}$$

$$= \frac{3}{\sqrt{t}} \cdot 2\sqrt{t} = \boxed{6}$$

Find the first and second derivatives and evaluate each at the given value of the parameter.

$x = \cos\theta,\ y = 3\sin\theta \qquad \theta = 0$

$$\frac{dy}{dx} = \frac{3\cos\theta}{-\sin\theta} \quad @\ \theta = 0 \rightarrow \frac{3\cos 0}{-\sin 0} = \frac{3}{0}$$

$\frac{dy}{dx}$ @ $\theta = 0$ is undefined

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left[\frac{3\cos\theta}{-\sin\theta}\right]}{-\sin\theta} = \frac{\frac{d}{d\theta}\left[-3\cot\theta\right]}{-\sin\theta}$$

$$= \frac{\cancel{-}3(-\csc^2\theta)}{\cancel{-}\sin\theta}$$

$$= -3\csc^3\theta$$

for $\theta = 0 \rightarrow \frac{d^2y}{dx^2} = \left(-3\csc(0)\right)^3$

is undefined.

Find the equation of the tangent at the specified parameter.

Need (x, y) & m

$$x = t - 1, \quad y = \frac{1}{t} + 1 \qquad \text{for } t = 1$$

$$x = 1 - 1 = 0$$

$$y = \frac{1}{1} + 1 = 2$$

$$(0, 2)$$

$$\text{slope} = \frac{dy}{dx} = \frac{\frac{-1}{t^2}}{1}$$

$$\text{slope} = \frac{dy}{dx} @ t = 1 \rightarrow \boxed{-1}$$

$$\boxed{y = -x + 2}$$

Purple Group Review

5) $y = \sqrt{x^2 - 4} - 2 \operatorname{arcsec} \frac{x}{2}$

$$y' = \frac{1}{2}(x^2 - 4)^{-1/2}(2x) - 2 \cdot \frac{1}{2} \cdot \frac{1}{\left|\frac{x}{2}\right| \sqrt{\frac{x^2}{4} - 1}}$$

$$y' = \frac{x}{\sqrt{x^2 - 4}} - \frac{1}{\frac{|x|}{2} \cdot \frac{\sqrt{x^2 - 4}}{\sqrt{4}}}$$

$$\frac{|x|}{|x|} \cdot \frac{x}{\sqrt{x^2 - 4}} - \frac{4}{|x|\sqrt{x^2 - 4}}$$

$$\frac{(x^2 - 4)^1}{|x|(x^2 - 4)^{1/2}}$$

$$\frac{\sqrt{x^2 - 4}}{|x|}$$

9) $$\lim_{\theta \to 0} \frac{\cos\theta \tan\theta}{\theta} = \frac{0}{0}$$

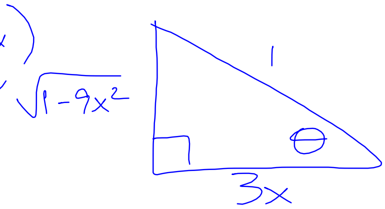
$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \frac{0}{0}$$

$$\lim_{\theta \to 0} = \frac{\cos\theta}{1} = 1$$

10) $$\lim_{x \to 0} \frac{\tan x}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sec^2 x}{1} = 1$$

11) $y = \sin(\underbrace{\arc\cos 3x}_{\theta})$



$y = \sin\theta$

$y = \sqrt{1-9x^2}$

$y' =$

HW: p. 693

1 - 27 odd, skip 17

HW Quiz on: p. 446
p. 467
p. 487